SYNOPSIS

Most of the fluids in industry unable to meet commonly accepted assumption of a linear relationship between the stress and the rate of strain and thus are characterized as non-Newtonian fluids. Due to complex rehology of biological fluids, different non-Newtonian fluids have been investigated and become more and more important due to their industrial applications. Polymer melts, certain oils and greases, clay coating and other suspensions, drilling mud are few examples of non-Newtonian fluids. Academic curiosity and practical applications have generated considerable interest in finding the solutions of differential equations governing the motion of these non-Newtonian fluids as such equations are highly non-linear as well as of higher order and hence they are more complicated than the second order Navier-Stokes equations. Calculations on non-Newtonian flow present a new challenge in flow analysis. Simulating these types of flows in order to calculate pipe and pump sizes presents a significant challenge to the engineers. Thus these fluids present some interesting challenges to researchers in engineering, applied mathematics and computer science.

Great diversity is found in the physical structure of non-Newtonian fluids and hence it is quite difficult to recommend a single constitutive equation to describe them. For this reason many non-Newtonian fluid model for constitutive equation have been proposed in literature [12,14, 34,35] and most of them are empirical or semi empirical. The present research on the flow problem for non-Newtonian fluid is hampered due to the lacking of proper classification of mathematical structure of a stress-strain relationship.

For the derivation of the constitutive equations governing the motion of non-Newtonian fluids, the mathematical structure of stress-strain relationship, which is non linear, is important in functional form. This relationship may be implicit or explicit. In the present thesis we have consider such relationship in the form of general arbitrary continuous function of the type

$$\Omega\,(\tau_{ij}\,,\,e_{ij}) = 0$$

The boundary layer flow occurs over various aerodynamic configurations such as wings, missiles, fuselage forms or channel flows. For the flow over wings and

channels, the boundary layer generally develops along a line on the surface (e.g. leading edge). The development of boundary layer is useful for efficient design procedures of both internal turbo machine components and external surface components. The gross allowances for the boundary layer were possible to make, using empirical relations along with results predicted from two dimensional boundary layer theory. But this method does not seem much promising for three dimensional boundary layer characteristics. As a result, a great deal of experimental and theoretical research on three dimensional flows has been carried on in recent years. But still a need exists for theoretical analyses which will lead to the predictions of boundary layer behavior.

Theoretical research has been restricted because of the complex nature of the equations describing the flow, while in a practical problems, the boundary layer is turbulent. Some success, however, has been achieved in the analysis of the three dimensional laminar equations. There is always a need to search for exact solutions of the laminar, incompressible, boundary layer equations for special types of main stream flows. From a physical point of view, these boundary layer equations have the capacity to admit a large number of invariant solutions which are known as similarity solutions. These invariant solutions are meant to reduce nonlinear partial differential equations of the boundary layer to a system of ordinary differential equations.

To investigate the non-Newtonian effects, the class of solutions known as similarity solutions plays an important role. This is because that is the only class of the exact solutions for the governing equations which are usually non-linear partial differential equations (PDEs) of the boundary layer type. Further this also serves as a reference to check approximate solutions. Now a day for similarity analysis many techniques are available, among them the similarity methods which invoke the invariance under the group of transformations are known as group theoretic methods. These methods are more recent and mathematically elegant hence they are widely used in different fields of science and engineering. The group theoretic methods involve mainly two different types of groups of transformations, namely, assumed group of transformations and deductive group of transformations. The linear group transformations, scaling group transformations,

spiral group transformations are the assumed group of transformations and are mainly due to Birkhoff [9] and Morgan [7]. Whereas the general group of transformation known as, deductive group of transformations can be further classify into two groups: finite group of transformations of Moran and Gaggioli [24] and infinitesimal group of transformation of Bluman and Cole [10], Bluman and Kumai [11].

Hansen and Herzig [1,2,3] has developed three dimensional boundary layer equations for Cartesian, Curvilinear and Polar coordinate system and derived similarity solution for all three cases. But all these past cases were limited to Newtonian fluids only. Schowalter [33], Hansen and Na [4,5] and Pakdemirli [22] and his coworkers have worked out similarity solutions for three dimensional boundary layer flows of non-Newtonian Power law fluids using scaling and spiral group of transformations. Timol and Kalthia [20] have carried out similarity analysis of three dimensional boundary layer equations of non-Newtonian fluids and integrated similarity equations for Reiner Philippoff fluids. Timol and Timol [19] probably were the first to establish boundary layer equations for steady three dimensional incompressible magneto fluid dynamic flows with pressure gradient and fluid injection. But in all these cases similarity equations were highly nonlinear coupled differential equations and remain numerically unsolved.

The main drawback of the similarity methods based on the assumed group of transformation at the outset of the analysis is that the resulting similarity solutions are restrictive and hence sometime leads to wrong conclusion that similarity transformations does not exist. On the other hand, the similarity methods based on general group of transformation are more systematic and lead to number of similarity solutions. Out of these the deductive group theoretic method provides a powerful tool because they are not based on linear operators, superposition, or any other aspects of linear solution techniques. Therefore, this method is applicable to nonlinear differential models.

The general group of transformation known as deductive group of transformation method has been successfully applied to various two dimensional non-linear flow problems by Abd-el- Malek et al [18], Parmar and Timol [13], Darji and Timol [26] and Adnan et al [15].

Literatures and papers on which the present analysis is based on finding similarity solution of partial differential equations, seldom give reasons and motivations for selecting transformations that reduce partial differential equations to an ordinary differential equations. Moreover, examination of the standard treatises on applied mathematics is usually of little help in providing information on similarity analysis. In view of these facts, we try to explore basic concept of Modern group theory and group transformation to derive the corresponding similarity transformation using deductive group method. Also, an attempt has been made to obtain numerical solution of reduced highly nonlinear differential equations. In addition, various methods that have been evolved are reviewed and evaluated. It is worth noting at this point that extended treatment of similarity analysis and general similarity methods for solving certain class of differential equations are found in a number of references. A summary of techniques for finding similarity solution of boundary value problem is very well explained by Hansen [6], Ames [32], Seshadri and Na [27], Bluman and Kumei [11].

The objective of the present thesis entitled " Group transformation treatment for engineering boundary value problems " is to derive fundamental basic boundary layer equations for both two-dimensional and three-dimensional non-Newtonian fluids past various external surfaces and numerical solution technique which can solve such a kind of problem of class of non-Newtonian fluids. The entire research work is sub-divided into eight chapters and description of individual chapter is given below:

Chapter 1

The literature review and general introduction of the fluids is outlined in 1st chapter. The brief explanation of the most frequently used terms in the present work like fluids, non-Newtonian fluid, incompressible fluid, laminar flow, boundary layer, Heat transfer and equations of continuity and motion are included in this chapter. Also, the relation between the stress tensor and the rate of deformation tensor for the different non-Newtonian fluid models like Power-law, Prandtl-Eyring, Williamson and Powel Eyring, Sutterby etc. are covered.

Chapter 2

This chapter deals with the theoretical background needed for all the subsequent chapters. The concept of the continuous transformation group is discussed in detail. The ideas of infinitesimal group of transformations and extended groups is introduced. The important concept of invariance of functions and differential equations under infinitesimal groups is elaborated. The existence of similarity transformation and similarity solutions are discussed . The advantages of group theoretic methods for solving highly nonlinear partial differential equations are highlighted. MATLAB has been used to solve higher order nonlinear ordinary differential equations and hence the description of bvp4c is included in this chapter.

Chapter 3

This chapter is focused on developing the deductive group-theoretic transformations for the similarity solution of steady, laminar, incompressible two dimensional boundary layer flow governing Williamson fluid. The applications of one parameter deductive group transformation are applied for simultaneous elimination of more than one variable. And consequently the system of governing highly non-linear partial differential equations with auxiliary conditions reduces to a non-linear ordinary differential equation with appropriate auxiliary conditions. The numerical solution for the considered Williamson fluid is derived systematically in dimensionless form as an application of engineering with MATLAB using MSABC.

Chapter 4

This chapter discusses an analysis made of the velocity and temperature distribution for the flow of a viscous incompressible fluid caused by the stretching of a sheet which issues in the Prandtl fluid. The governing partial differential equations are reduced to ordinary differential equations using deductive group transformation and similarity solution is derived. Numerical solutions to the reduced non-linear similarity equations are then obtained by adopting Runge-Kutta and shooting method using the Nachtsheim- Swigert iteration technique with MATLAB. The results of the numerical solution are then

presented graphically in the form of velocity and temperature profiles. The corresponding skin friction coefficient and the Nusselt number are also calculated and displayed in table showing the effects of various parameters on them.

Chapter 5

In the present chapter, using general group theoretic method, the similarity transformations are derived for steady, laminar, incompressible two dimensional natural convection boundary layer flow of non-Newtonian Sutterby fluid past a vertical isothermal plate. The application of a one-parameter group reduces the number of independent variables by one and consequently the system of governing highly non-linear partial differential equations with auxiliary conditions reduces to a non-linear ordinary differential equation with appropriate auxiliary conditions. Numerical solutions to the reduced highly non-linear similarity equations are then obtained by adopting Runge-Kutta and shooting method using the Nachtsheim-Swigert iteration technique with MATLAB. The numerical solution for the considered Sutterby fluid is derived for different of flow consistency index A systematically in dimensionless form. Further the graph analysis is made of the velocity and temperature distribution in terms of dimensionless parameters and hence it represents behavior of Sutterby fluid.

Chapter 6

This chapter deals with general group theoretic method which is employed to investigate the similarity transformations for steady, laminar, incompressible two dimensional mixed convection MHD boundary layer flow of non-Newtonian Sutterby fluid. The purpose of investigating the similarity transformations is to make use of group methods to represent the problem in the form of an ordinary differential equation with appropriate auxiliary conditions. Numerical integrations to the reduced non-linear coupled similarity equations are then carried out using Runge-Kutta method with MATLAB. Numerical solution obtained for the considered Sutterby fluid is presented graphically for various flow consistency index A. It should be noted that for

A=0, the Sutterby model reduces to Newtonian and for A=1, to that of Prandtl Eyring model respectively.

Chapter 7

The work considered in this chapter is focused on deductive group-theoretic transformations to develop the similarity solution of steady, laminar, incompressible quasi three dimensional boundary layer flow governing power law fluid. The application of one-parameter group reduces the number of independent variables to one and consequently the system of governing, highly non-linear partial differential equations reduces to a self similar, non-linear ordinary differential equation with appropriate auxiliary conditions. The numerical solution for a power law fluid considered for small cross flow is obtained systematically with the help of MATLAB using MSABC in dimensionless form.

Chapter 8

The current chapter is an outcome of continuing study of ways in which partial differential equations (PDES) associated with the problems of physical interest may be simplified through transformation of independent and dependent variables. We have suggested the general deductive group theoretic method based on an invariance feature of PDES, in a systematic manner, so that this investigation leads to similarity transformations which transforms given problem into a simpler problem in some sense or other. The method is employed to steady, laminar, incompressible three dimensional boundary layer flow governing non-Newtonian Sutterby fluid - a model which is mathematically more complex. The transformed similarity equations are of third order and highly non-linear in nature. The numerical solution is obtained to illustrate the simplicity of the attempt made. The effect of governing parameters are demonstrated with the help of graphs for heat transfer. The problem is studied theoretically. An experimental verification will greatly enhance the value of present study.

Thus in the present thesis we have made an attempt to employ ` the deductive group theoretic method to simplify boundary value problem arising in engineering and also derived numerical solution for various kinds of non-linear differential

equations governing the boundary layer flow of two and three dimensional Non-Newtonian Fluids using MATLAB.

Through the present investigation an humble endeavor to show that the technique used is quite promising and the results so obtained will serve as a good approximation to the physical model of interest in predicting the qualitative fluid behavior . Further extension is wide open for more complicated real life problem of various field like aerospace, Bio-fluid mechanics, magneto hydrodynamics, medical science etc. Method is suitable for undergraduate students, graduate students and researchers in the area of engineering and applied science.

It is worth to mention that a pan of the subject matter of the present investigation has been published /communicated in the form of research papers:

1. Invariance Analysis of Williamson Model Using the Method of Satisfaction of Asymptotic Boundary Conditions - International Journal of Mathematics and Scientific Computing Vol.4, No.2, 2014 pp. 74-78.

2. Similarity solution of natural convection boundary layer flow of non - Newtonian Sutterby fluids – International Journal of Advances in Applied Mathematics and Mechanics 2(2), 2014 , pp. 150-158.

3. Similarity Solutions of quasi three dimensional Power law fluids using the Method of Satisfaction of Asymptotic Boundary Conditions -Accepted for publication and article in press has come in Alexandria Engineering (Elsevier) Journal .

4. Prandtl Fluid Model Over Stretching Sheet - International Journal of Applied Mathematics and Mechanics (communicated).

5. Deductive group theoretic analysis of mixed convection MHD Sutterby fluids - Journal of Advanced Mathematics and Applications (communicated).

6. Similarity Solutions of quasi three dimensional non-Newtonian Sutterby fluids – Mathematics Today (communicated).

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